PERGAMON

International Journal of Solids and Structures 37 (2000) 6051–6062

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

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The transient response of a piezoelectric strip with a vertical crack under electromechanical impact load

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Received 8 December 1998; in revised form 7 September 1999

Abstract

In this paper, the dynamic electromechanical response of a piezoelectric strip with a central crack vertical to the boundary was investigated. Based on the superposition principle and integral transform techniques, the present problem was reduced to the solution of two pairs of dual integral equations. To accommodate the finite size of the strip, two different Fourier transforms, about the x and y coordinates, were assumed. The solution was obtained in the Laplace transformed domain in terms of Fredholm integral equations of the second kind by a modified Copson's method. The Laplace inversion was then carried out to obtain the resulting dynamic stress and electric displacement intensities. Unlike the findings observed in the static fracture behavior of piezoelectric materials, the present study indicates that the dynamic electric field will retard or promote the crack propagation at the different loading stages. Furthermore, the dynamic electric response is proportional to the electric impact and is independent of the applied mechanical impact. It is also shown that both the mechanical and electric response around the crack tip are greatly influenced by the free boundaries of the piezoelectric strip. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Strip; Transient; Piezoelectric; Impact; Electromechanical

1. Introduction

In view of their brittleness, piezoelectric materials have a tendency to develop critical cracks during the manufacturing and the poling processes. The existence of these defects will greatly affect the mechanical integrity and electromechanical behavior of this class of materials (Jain and Sirkis, 1994). Although so much attention has been focused on the static fracture analysis of piezoelectric materials,

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PII: S0020-7683(99)00259-0

less attention has been paid to the study of the corresponding dynamic problem. This may be due to the complexity of the mathematical treatment and may be due to the presence of complex electromechanical coupling terms, especially when the finite size of the strip is taken into account. It is for this reason that we offer the current study.

It is worth noting, however, that a number of researchers have contributed to the steady state dynamic response of a cracked piezoelectric material. Shindo and Ozawa (1990) first investigated the steady state dynamic response of a cracked piezoelectric material under the action of incident plane harmonic waves. The dynamic Green's functions for anisotropic piezoelectric materials were derived by Norris (1994). The results were only presented in the time transform domain, because of the complexity associated with the time inversion. Khutoryansky and Sosa (1995) proposed the dynamic representation formulas and fundamental solutions for piezoelectricity. Shindo et al. (1996) studied the dynamic response of a cracked dielectric medium under the action of harmonic waves in a uniform electric field. In their most recent work, Narita and Shindo (1998a) investigated the scattering of Love waves by a surface-breaking crack in a piezoelectric layer over an elastic half plane. Meguid and Wang (1998) investigated the dynamic anti-plane interaction of two cracks in piezoelectric medium under incident antiplane shear wave loading with the conducting crack assumption.

In engineering applications, piezoelectric materials may experience transient dynamic loads as well as steady harmonic loads. It is, therefore, of great importance to investigate the transient dynamic response of cracked piezoelectric materials. Li and Mataga (1996) studied the problem of a semi-infinite crack propagating in an infinite piezoelectric material. They focused their attention on the effect of the propagating velocity of the crack on the dynamic response of the electromechanical field around the crack tip. In their work, the transient dynamic electromechanical loads were taken into consideration and a new surface wave was reported.

The electric boundary condition along the crack faces is still an open problem. Generally, there are two well-accepted electric boundary conditions, namely: the permeable and impermeable boundary conditions. In an investigation by McMeeking (1989), he concluded that the concentration of both the mechanical and electric fields are governed by a parameter $(\varepsilon_f/\varepsilon_m)(b/a)$, where b/a is the aspect ratio of an elliptic crack, and ε_f and ε_m are the respective dielectric coefficients of the medium in that crack (i.e., the permittivity of air or vacuum in general) and the piezoelectric matrix. He further argued that as long as $\varepsilon_f/\varepsilon_m$ is less than one-tenth of b/a and $b/a \ll 1$, the impermeable crack solution serves as a good approximation to the problem. Pak and Tobin (1993) indicated that the results for an elliptic inclusion in piezoelectric materials will reduce to those of an impermeable crack problem as the inclusion approaches a crack. Some researchers such as Pak and Goloubeva (1996), Zhao et al. (1997), Liu et al. (1998) and Qin and Mai (1999) use the impermeable condition, while others disagree with this condition, including Zhang et al. (1998) and Narita and Shindo (1998b, 1998c). Furthermore, recent experimental research by the authors using lead lanthanum zirconate titanate (PLZT) showed arcing across the interface, indicating an impermeable interface condition. It is also worth noting that the most recent work on the static fracture behavior of piezoelectric materials (Gao and Fan, 1999) had shown that the stress field intensity using vacuum boundary condition, outlined by Li and Mataga (1996), is the same as that obtained based upon the impermeable crack assumption.

In the present work, we develop a new model to treat the dynamic fracture behavior of a vertical crack embedded in a finite width piezoelectric strip. Impermeable condition is adopted in the present study and attention is devoted to transient response and the boundary effects on the dynamic stress and electric displacement intensities, which is rarely observed in the literature. By employing integral transform techniques and the superposition principle, the problem is reduced to two pairs of dual integral equations which are solved in terms of Fredholm integral equations of the second kind.

2. Formulation of the problem

Consider a piezoelectric strip of width $2h$ containing a Griffith crack of length $2a$ with remote antiplane mechanical and inplane electric impacting loads acting on it, as depicted in Fig. 1. The electromechanical impact is a Heaviside step function of time, i.e., $\tau_{zy}(t) = \tau_0 H(t)$; $D_y(t) = D_0 H(t)$. With the aid of the superposition principle, the present problem can be decomposed into two problems: (a) uniform electromechanical impact on a crack-free piezoelectric strip with non singular behavior, and (b) accompanying electromechanical impact loads $\tau_{zy}(t) = -\tau_0 H(t)$ and $D_y(t) = -D_0 H(t)$ acting on the surfaces of the crack in a piezoelectric strip. Details of problem (b) are given below.

A set of Cartesian coordinate (x, y, z) is attached to the center of the crack. The x-axis is directed along the crack line and y-axis is perpendicular to it. The poled piezoelectric ceramic strip, with z-axis being the poling direction, occupies the region $(-h < x < h, -\infty < y < \infty)$, and is thick enough in that direction to allow a state of antiplane strain. Because of the assumed symmetry in geometry and loading, it is sufficient to consider the problem only in the regime $0 < x < h$ and $0 < y < \infty$.

The piezoelectric boundary value problem for anti-plane shear is simplified if we consider only the out-of-plane displacement and the inplane electric displacement, such that the constitutive equation can

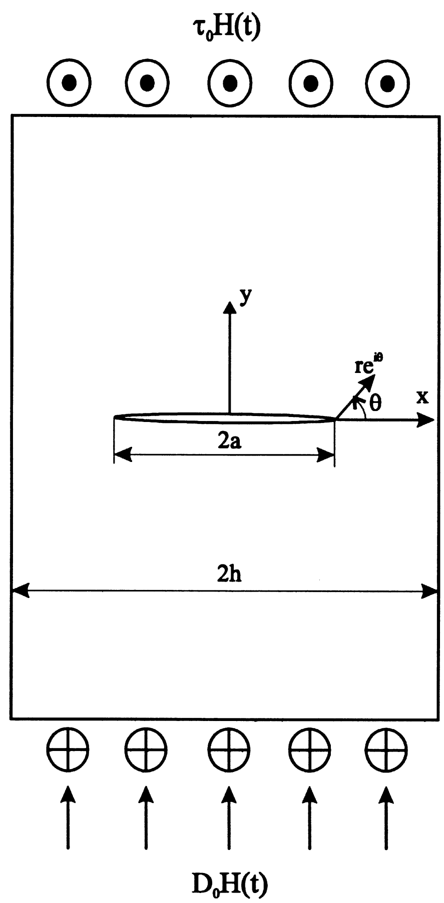

Fig. 1. A piezoelectric strip with a vertical crack under antiplane electromechanical impact load.

be written as (Parton, 1976)

$$\tau_{zj} = c_{44} w_{,j} + e_{15} \phi_{,j} \tag{1}$$

$$D_j = e_{15} w_{,j} - \varepsilon_{11} \phi_{,j} \tag{2}$$

where τ_{zj}, D_j $(j = x, y)$ are the antiplane shear stress and inplane electric displacement; c_{44}, e_{15}, and ε_{11} are the shear modulus, piezoelectric coefficient and dielectric parameter; w and ϕ are the mechanical displacement and electric potential, respectively.

The dynamic antiplane governing equations for piezoelectric materials are

$$c_{44} \nabla^2 W + e_{15} \nabla^2 \phi = \rho \partial^2 w / \partial t^2 \tag{3}$$

$$e_{15} \nabla^2 w - \varepsilon_{11} \nabla^2 \phi = 0 \tag{4}$$

In the above equations, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator and ρ is the mass density of the piezoelectric material.

Substituting Eq. (4) into Eq. (3), we can obtain the wave equation in piezoelectric materials, viz.

$$\nabla^2 w = c_2^{-2} \partial^2 w / \partial t^2 \tag{5}$$

in which,

$$c_2 = \sqrt{\mu/\rho}; \quad \mu = c_{44} + e_{15}^2/\varepsilon_{11}. \tag{6}$$

Let us now assume that the two edges of the strip are stress and electric displacement free, and that the electromechanical impact acts suddenly on the surface of the cracks at $t = 0$, so that the boundary condition can be expressed as follows:

$$\tau_{zx}(h, y, t) = D_x(h, y, t) = 0, \quad -\infty < y < \infty$$

$$\tau_{zy}(x, 0, t) = -\tau_0 H(t), \quad 0 < x < a$$

$$D_y(x, 0, t) = -D_0 H(t), \quad 0 < x < a$$

$$w(x, 0, t) = \phi(x, 0, t) = 0, \quad a \leq x < h \tag{7}$$

It should be noted that the electrically impermeable boundary condition of the crack is adopted in Eq. (7). This boundary condition has been used by numerous other investigators, including: McMeeking (1989), Pak (1990), Suo et al. (1992), Sosa (1992), Zhang and Hack (1992), Pak and Goloubeva (1996), Zhao et al. (1997), Liu et al. (1998), Qin and Mai (1999), among others. The electric boundary condition along the free edges of the strip, expressed by the first equality of (7), can be readily obtained from the assumption that there is no free charge at any surface of this piezoelectric strip, as Shindo et al. (1998) did recently for the problem of a piezoelectric half space.

3. Dynamic description of the electromechanical field

Define a pair of Laplace transforms by the equations:

$$f^*(p) = \int_0^\infty f(t)\exp(-pt)\,\mathrm{d}t, \quad f(t) = \frac{1}{2\pi i}\int_{Br} f^*(p)\exp(pt)\,\mathrm{d}p \tag{8}$$

in which "Br" stands for the Bromwich path of integration. The time-dependency in Eqs. (4) and (5) are eliminated by the application of Eq. (8). Depending on the evenness and oddness of the functions w and ϕ in the variables x and y, the Fourier cosine and sine transforms are then applied, resulting in:

$$w^*(x, y, p) = \frac{2}{\pi}\left[\int_0^\infty A(s, p)\exp(-\gamma y)\cos(sx)\,\mathrm{d}s + \int_0^\infty B(s,p)\cosh(\gamma x)\sin(sy)\,\mathrm{d}s\right] \tag{9}$$

$$\phi^*(x, y, p) = \frac{e_{15}}{\varepsilon_{11}}w^*(x, y, p) + \psi^*(x, y, p) \tag{10}$$

with

$$\psi^*(x, y, p) = \frac{2}{\pi}\left[\int_0^\infty C(s, p)\exp(-sy)\cos(sx)\,\mathrm{d}s + \int_0^\infty D(s, p)\cosh(sx)\sin(sy)\,\mathrm{d}s\right] \tag{11}$$

in which

$$\gamma(s, p) = \sqrt{s^2 + p^2 c_2^{-2}} \tag{12}$$

Consequently, the Laplace transform of Eqs. (1) and (2) can be written as

$$\tau^*_{zj} = \mu w^*_{,j} + e_{15}\psi^*_{,j} \tag{13}$$

$$D^*_j = -\varepsilon_{11}\psi^*_{,j} \tag{14}$$

The Laplace transform of the boundary condition yields:

$$\tau^*_{zx}(h, y, p) = D^*_x(h, y, p) = 0, \quad -\infty < y < \infty$$

$$\tau^*_{zy}(x, 0, p) = -\tau_0/p, \quad 0 < x < a$$

$$D^*_y(x, 0, p) = -D_0/p \quad 0 < x < a$$

$$w^*(x, 0, p) = \phi^*(x, 0, p) = 0, \quad a \le x < h \tag{15}$$

Substituting Eqs. (9) and (10) into Eqs. (13) and (14), and the resulting expressions into Eq. (15), we can obtain two pairs of dual integral equations for the two unknown functions $A(s, p)$, and $C(s, p)$:

$$\frac{2}{\pi}\int_0^\infty A(s, p)\cos(sx)\,\mathrm{d}s = 0 \quad a < x < h$$

$$\frac{2}{\pi}\int_0^\infty \gamma A(s,p)\cos(sx)\,\mathrm{d}s = \frac{2}{\pi}\int_0^\infty sB(s,p)\cosh(\gamma x)\,\mathrm{d}s + (\tau_0 + e_{15}D_0/\varepsilon_{11})/(\mu p) \quad 0<x<a \tag{16}$$

and

$$\frac{2}{\pi}\int_0^\infty C(s,p)\cos(sx)\,\mathrm{d}s = 0 \quad a<x<h$$

$$\frac{2}{\pi}\int_0^\infty sC(s,p)\cos(sx)\,\mathrm{d}s = \frac{2}{\pi}\int_0^\infty sD(s,p)\cosh(sx)\,\mathrm{d}s - D_0/(\varepsilon_{11},p) \quad 0<x<a \tag{17}$$

From the boundary conditions of the first expression in (15), we can obtain the relationship between $A(s)$ and $B(s)$, and $C(s)$ and $D(s)$, such that

$$B(s,p) = \frac{2s}{\pi\gamma\sinh(\gamma h)}\int_0^\infty A(\eta,p)\frac{\eta}{\eta^2+\gamma^2}\sin(\eta h)\,\mathrm{d}\eta \tag{18}$$

and

$$D(s,p) = \frac{2}{\pi s\sinh(sh)}\int_0^\infty C(\eta,p)\frac{\eta^2}{\eta^2+s^2}\sin(\eta h)\,\mathrm{d}\eta \tag{19}$$

By substituting Eq. (18) into (16), we can finally obtain the solution of $A(s)$ by means of a modified Copson's (Copson, 1961) method

$$A(s,p) = \frac{\pi a^2}{2\mu p}(\tau_0 + e_{15}D_0/\varepsilon_{11})\int_0^1 \sqrt{\xi}\varphi_3^*(\xi,p)J_0(sa\xi)\,\mathrm{d}\xi \tag{20}$$

where $J_0\ (\cdot)$ is the zero-order Bessel function of the first kind, while the unknown function $\varphi_3^*(\xi,p)$ is determined by the following Fredholm integral equation of the second kind

$$\varphi_3^*(\xi,p) + \int_0^1 K(\xi,\eta,p)\varphi_3^*(\eta,p)\,\mathrm{d}\eta = \sqrt{\xi} \tag{21}$$

The symmetric kernel of the integral equation (21) is given by

$$K(\xi,\eta,p) = (\xi\eta)^{1/2}\int_0^\infty \left\{ \left[a\gamma(s/a,p) - s\right]J_0(s\xi)J_0(s\eta) - \frac{s^2\exp\left[-h\gamma(s/a,p)\right]}{a\gamma(s/a,p)\sinh\left[h\gamma(s/a,p)\right]} I_0\left[a\xi\gamma(s/a,p)\right.\right.$$
$$\left.\left.\times\right] I_0\left[a\eta\gamma(s/a,p)\right]\right\}\mathrm{d}s \tag{22}$$

with $\gamma(s/a,p)$ being

$$\gamma(s/a,p) = \sqrt{(s/a)^2 + p^2 c_2^{-2}} \tag{23}$$

In Eq. (22), $I_0\ (\cdot)$ is zero-order modified Bessel function of the first kind.

Similarly, the solution of the dual integral equation (17) can be written as follows

$$C(s,p)=\frac{\pi a^2 D_0}{2\varepsilon_{11}p}\int_0^1\sqrt{\xi}\chi(\xi)J_0(sa\xi)\,\mathrm{d}\xi \tag{24}$$

with $\chi(\xi)$ being determined by the following Fredholm integral equation of the second kind

$$\chi(\xi)+\int_0^1\vartheta(\xi,\eta)\chi(\eta)\,\mathrm{d}\eta=\sqrt{\xi} \tag{25}$$

The symmetric kernel $\vartheta(\xi,\eta)$ of the integral Eq. (25) is given by

$$\vartheta(\xi,\eta)=-(\xi\eta)^{1/2}\int_0^\infty\frac{2s}{\exp(2sh/a)-1}I_0(s\xi)I_0(s\eta)\,\mathrm{d}s \tag{26}$$

Substituting Eqs. (20) and (24) into Eqs. (18) and (19), we can finally determine all the four unknown functions $A(s,p)$, $B(s,p)$, $C(s,p)$ and $D(s,p)$. This will allow the determination of the singular parts of the dynamic stress and electric displacement in the neighborhood of the crack tip.

4. Intensities of stress and electric displacement

The dynamic stress and electric field can be obtained by determining the inverse of the Laplace transform of the stress and electric displacement expressions. From the point of view of fracture mechanics, however, only the singular stress near the crack tip will be derived here. The integral expression for the Laplace transform of the stress and electric displacement can be obtained by substituting Eqs. (18)–(20) and (24) into Eqs. (9) and (10), and the resulting expressions into Eqs. (13) and (14). The divergence of the integral near the crack tip corresponds to the behavior of the intgrand as the integration variable s tends to infinity. The portions of $A(s,p)$ and $C(s,p)$ that contribute to the singular behavior are found from the integrals of Eqs. (20) and (24) by parts:

$$A(s,p)=\frac{\pi a}{2\mu sp}(\tau_0+e_{15}D_0/\varepsilon_{11})\varphi_3^*(1,p)J_1(as)+\cdots,$$

$$C(s,p)=-\frac{\pi a D_0}{2\varepsilon_{11}sp}\chi(1)J_1(as)+\cdots$$

If we now let the crack tip be the origin of the polar coordinate system, shown in Fig. 1, then

$$r\exp(i\theta)=x-a+iy \tag{27}$$

where $i=\sqrt{-1}$. From the above results, the stress and electric displacement around the crack tip can be expressed as

$$\tau_{zy}+i\tau_{zx}=\frac{K_3^\tau(t)}{\sqrt{2\pi r}}\exp(-i\theta/2)+O(r) \tag{28}$$

$$D_y+iD_x=\frac{K_3^D(t)}{\sqrt{2\pi r}}\exp(-i\theta/2)+O(r) \tag{29}$$

where the intensity factors of stress and electric displacement $K_3^\tau(t)$and $K_3^D(t)$ can be expressed as follows

$$K_3^\tau(t) = \left[(\tau_0 + e_{15}D_0/\varepsilon_{11})M(t) - e_{15}D_0H(t)\chi(1)/\varepsilon_{11}\right]\sqrt{\pi a} \tag{30}$$

$$K_3^D(t) = D_0H(t)\chi(1)\sqrt{\pi a} \tag{31}$$

with

$$M(t) = \frac{1}{2\pi i}\int_{Br} \frac{\varphi_3^*(1,p)}{p}\exp(pt)\,\mathrm{d}p \tag{32}$$

where $\varphi_3^*(1,p)$ and $\chi(1)$ are determined by Eqs. (21) and (25), respectively.

5. Numerical examples and discussions

The determination of the intensities of stress and electric displacement requires the solution of Fredholm integral equations (21) and (25), respectively. In the present work, the solution of these integral equations was carried out using the method described by Fan (1991), wherein, the improper integral over s is expanded in Gauss–Legendre quadrature points and weights. To obtain the dynamic stress intensity factor in the physical plane, we used the method outlined by Miller and Guy (1966).

By examining Eq. (31), one can see clearly that the dynamic electric displacement intensity factor is proportional to the applied dynamic electric load and is independent of the applied mechanical load.

Fig. 2 shows the variation of the normalized electric displacement intensity factor $[K_3^D(t)/(D_0(\pi a)^{1/2})]$ with a normalized crack length (a/h). The figure demonstrates clearly that initially the electric displacement intensity factor develops slowly with the increase in the crack length up to $a/h = 0.4$. Beyond that length, the electric displacement intensity increases rapidly with the increase of a/h, tending to infinity as a/h approaches unity.

In the transient response computations shown in Fig. 3, a normalized electric load, $\lambda = (e_{15}D_0/\varepsilon_{11})/\tau_0$, is introduced with assigned values 0.1, 0.2, 0.5, 1.0, or 2.0, respectively. These values were selected based upon the experimental data of Park and Sun (1995) for PZT. The results of the transient response of the normalized dynamic stress intensity factor (DSIF), $K_3^\tau(t)/(\tau_0\sqrt{\pi a})$ versus normalized time (c_2t/a) at $\lambda =$

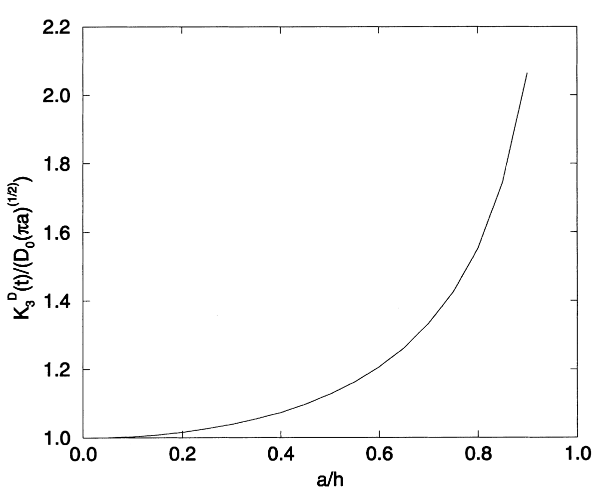

Fig. 2. Normalized dynamic electric displacement intensity factors versus normalized crack lengyh a/h.

0.1 and 2.0 are presented in Fig. 3(a) and (b), respectively. Fig. 3(a) shows that the higher the ratio of the crack length to the width of the piezoelectric strip, the higher the peak value of the DSIF, while the dynamic mechanical loads dominates the peak value of DSIF. Fig. 3(b), on the other hand, shows that while the dynamic electric load dominate the peak value of the DSIF, an inverse relation exists for the case where a/h is less than 0.8. Beyond this value, the higher a/h, the higher the peak value of DSIF. These two figures also show the significant effect of the boundary on the resulting response for different normalized electric loads (λ).

To illustrate the influence of the dynamic electric load on the propagation of a crack in a piezoelectric material, we have plotted the normalized DSIF versus normalized time as a function of the electric load. The results for $a/h = 0.3$ and 0.9 are shown in Fig. 4(a) and (b). From these figures, it can be clearly

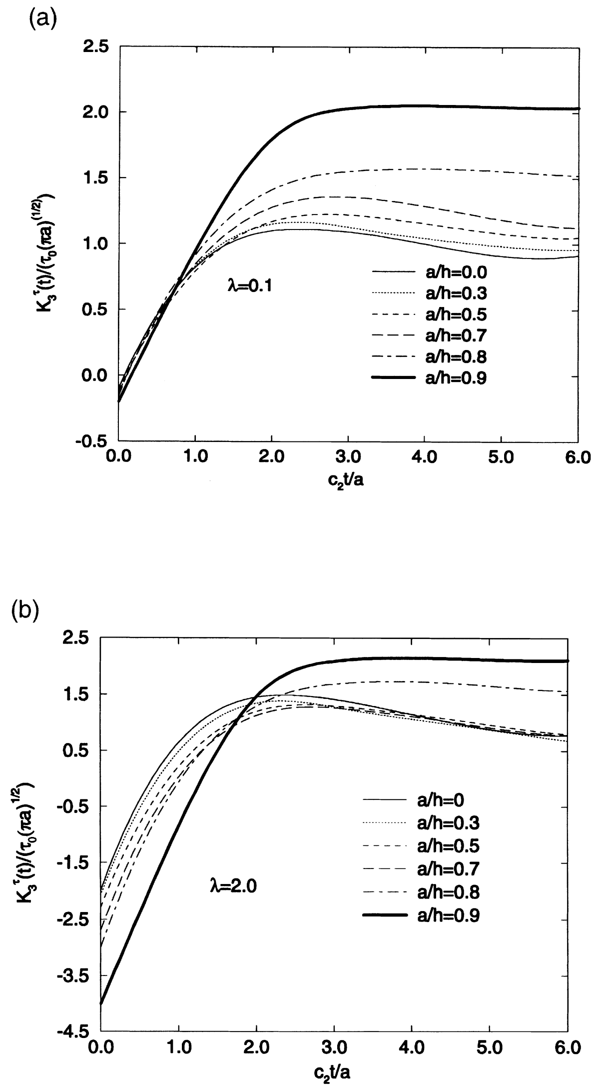

Fig. 3. Normalized DSIF versus normalized time for different normalized crack length a/h and normalized electric loads: (a) $\lambda = 0.1$, and (b) $\lambda = 2.0$.

seen that the presence of the dynamic electric field will retard the propagation of the crack at the very beginning of the impact process. However, after the normalized time exceeds a certain value, say 1.5 or so, the higher values of λ lead to higher values of DSIF. This continues until the normalized time exceeds 3.5–4.0. Beyond that time, the higher values of λ lead to higher values of DSIF resulting from boundary effects ($a/h = 0.9$). In the case where $a/h = 0.3$, the higher values of λ lead to lower values of DSIF. Therefore, one can conclude that at the different stages of the loading process, the presence of the electric field will promote or retard the propagation of the crack in piezoelectric materials depending upon the time elapsed and the crack length a/h.

An interesting situation arising from our work is that Eq. (30) indicates that the DSIF provided accounts for both the electric and mechanical effects. It is worth noting, however, that the work of Pak (1990) and Park and Sun (1995) for quasi-static fracture mechanics indicates that the electric load alone

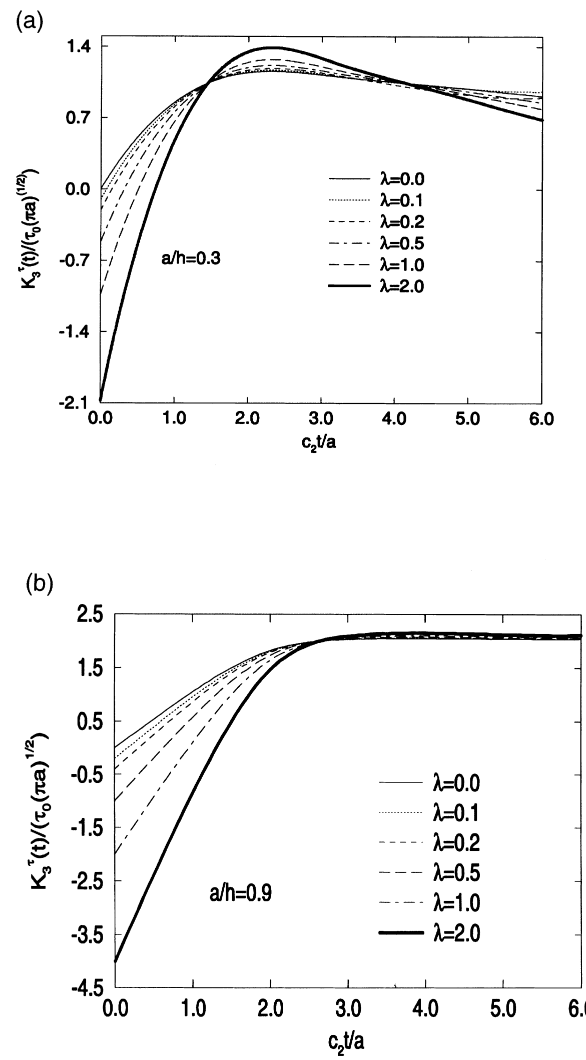

Fig. 4. Normalized DSIF versus normalized time for different normalized electric loads and crack lengths a/h: (a) $a/h = 0.3$, and (b) $a/h = 0.9$.

cannot produce stress intensity factors at the crack tip. This implies that the stress intensity factor, in their case, cannot be used as a fracture criterion because of its independence on the electric field.

6. Concluding remarks

In this article, the transient response of a cracked piezoelectric strip of a finite width under dynamic electromechanical loads is investigated. The dynamic intensities of stress and electric displacement have been given in terms of the Fredholm integral equations of the second kind. Numerical computation of the resulting coupled dynamic equations reveals that the electric field will retard or promote the propagation of the crack in a piezoelectric strip, at different stages of the impact loading process, depending upon the crack length (a/h) and the electric load (λ).

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